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320. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A heavy uniform chain of length l is hung over a rough horizontal cylinder of radius r. Show that one end of the chain will be $[2\mu r/(\mu^2 + 1)](e^{\mu\pi} + 1) + l(e^{\pi\mu} - 1)$ units lower than the other, just when the chain begins to move, the coefficient of friction being μ.

SOLUTION BY WILLIAM HOOVER, Columbus, Ohio.

Let x = the longest part hanging down, and $l - (\pi r + x)$ = the shortest, θ being the angle which the radius through any point of the string makes with the vertical diameter and positive in the direction left to right; and T, T' the tensions at any points in those parts of the string to which x and $l - (\pi r + x)$ belong. Then if C be a constant, we have

$$T = Ce^{\mu\theta} + \frac{r}{\mu^2 + 1}\{2\mu \sin\theta + (1 - \mu^2)\cos\theta\}, \tag{1}$$

by theory, μ and θ being both negative, the string being supposed to move in the direction of x.

When $\theta = -\pi/2$, $\mu = -\mu$, and $T = x$, a unit of length of string weighing a unit, we have

$$C = \left(x - \frac{2\mu r}{\mu^2 + 1}\right)e^{-(1/2)\pi\mu}, \tag{2}$$

and (1) then becomes

$$T = \left(x - \frac{2\mu r}{\mu^2 + 1}\right)e^{[\theta-(\pi/2)]\mu} + \frac{r}{\mu^2 + 1}\{-2\mu \sin\theta + (1 - \mu^2)\cos\theta\}. \tag{3}$$

When $\theta = 0$,

$$T = \left(x - \frac{2\mu r}{\mu^2 + 1}\right)e^{-(1/2)\pi\mu} + \frac{r(1 - \mu^2)}{\mu^2 + 1}, \tag{4}$$

which is the tension at the vertex.

Again, $T' = l - (\pi r + x)$, when $\theta = \pi/2$, $\mu = -\mu$; then

$$Ce^{-(1/2)\pi\mu} - \frac{2\mu r}{\mu^2 + 1} = l - (\pi r + x),$$

or

$$C = \left\{l - (\pi r + x) + \frac{2\mu r}{\mu^2 + 1}\right\}e^{(1/2)\pi\mu}. \tag{5}$$

Hence

$$T' = \left\{l - \pi r - x + \frac{2\mu r}{\mu^2 + 1}\right\}e^{\mu[\theta+(\pi/2)]} + \frac{r}{\mu^2 + 1}\{-2\mu \sin\theta + (1 - \mu^2)\cos\theta\}. \tag{6}$$

When $\theta = 0$,

$$T' = \left\{l - \pi r - x + \frac{2\mu r}{\mu^2 + 1}\right\}e^{(1/2)\pi\mu} + \frac{r(1 - \mu^2)}{\mu^2 + 1}. \tag{7}$$

For equilibrium, $T = T'$, and then

$$\left(l - \pi r - x + \frac{2\mu r}{\mu^2 + 1}\right)e^{(1/2)\pi\mu} = \left(x - \frac{2\mu r}{\mu^2 + 1}\right)e^{-(1/2)\pi\mu}, \tag{8}$$

giving

$$x = \frac{l - \pi r}{1 + e^{-\pi\mu}} + \frac{2\mu r}{\mu^2 + 1}, \tag{9}$$

and

$$l - (\pi r + x) = \frac{(l - \pi r)e^{-\pi\mu}}{1 + e^{-\pi\mu}} - \frac{2\mu r}{\mu^2 + 1}. \tag{10}$$

(9) and (10) give the two vertical parts of the string, their difference being

$$\frac{(e^{\pi\mu} - 1)(l + \pi r)}{1 + e^{-\pi\mu}} + \frac{4\mu r}{\mu^2 + 1},$$

not agreeing with the required difference in the statement of the problem.

Also solved with same result by PAUL CAPRON, RALPH E. ROOT, and ELIJAH SWIFT.